

PENDULUM AUTOGRAPHS

II.

IT naturally occurred to try what would be the effect of introducing a third joint in the suspension of the pendulum, and altering the angles between the different joints. First I tried three joints at angles of 60° , by means of two intermediate pieces attached to the rod, to the cross-bar,

and to one another, by string-hinges. The result did not differ from that obtained with two joints. This surprised me, for I had expected that each joint in taking its share of the swing would somehow assert its claim to its own proper period distinct from the periods of the other two. But on close examination, attaching long slender splints of wood to the cross-bar, the two intermediaries, and the rod, and bringing their depending ends near together, by

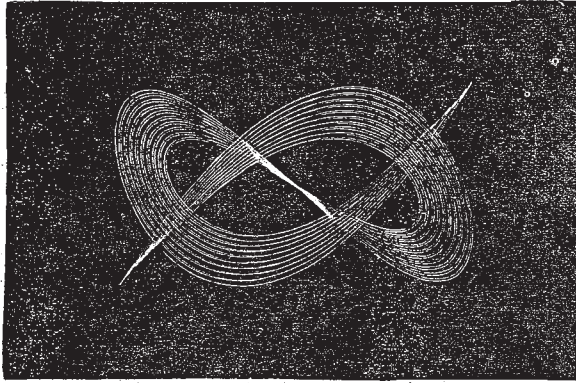


FIG. 7.—Proportion 3 : 5.—Cusped type.

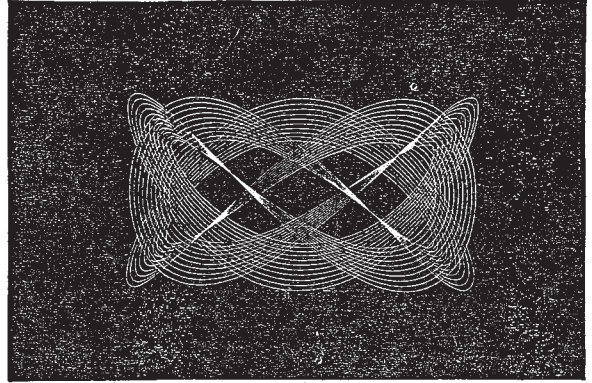


FIG. 8.—Proportion 3 : 5.—Looped type.

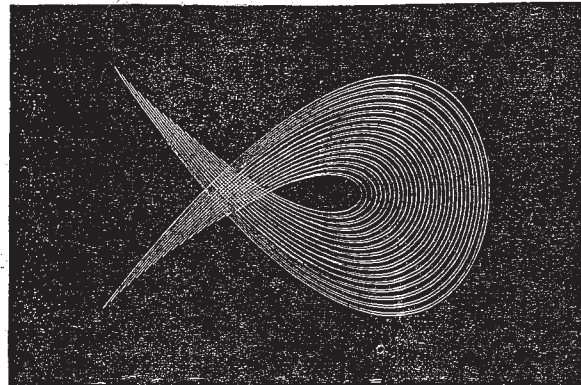


FIG. 9.—Proportion 2 : 3.—Cusped type.

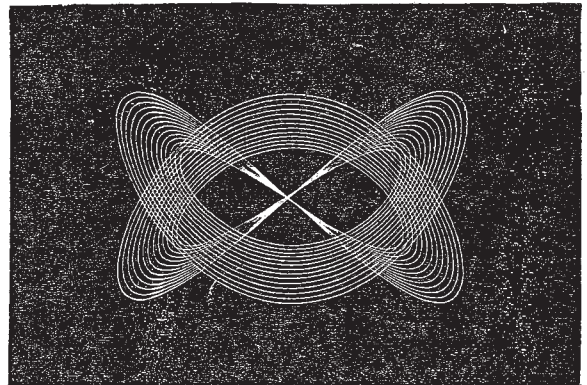


FIG. 10.—Proportion 2 : 3.—Looped type.

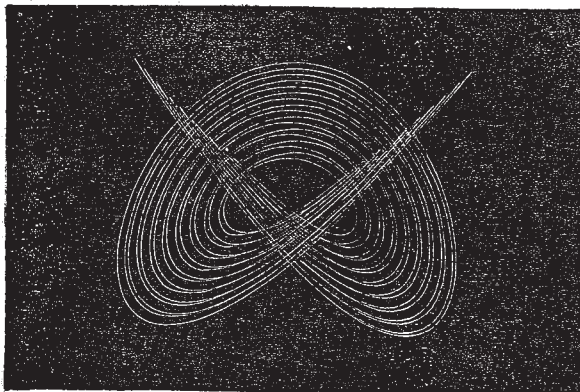


FIG. 11.—Proportion 3 : 4.—Cusped type.

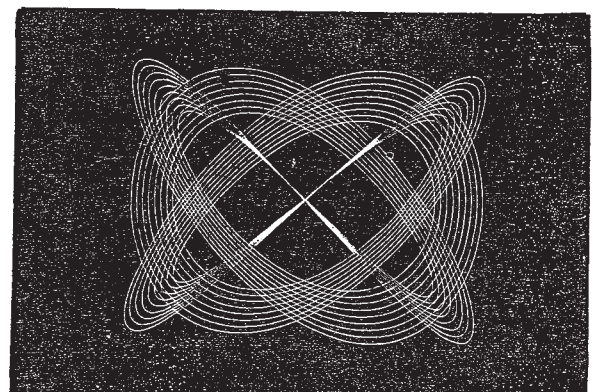


FIG. 12.—Proportion 3 : 4.—Looped type.

pairs, for comparison of relative motions, I found that the two upper joints alone were able to make good their title to independent periods of swing: the third joint acted merely as subsidiary to the second, merely serving to facilitate the bend at the second joint, and always synchronous with that bend, never able to establish a proper period of its own. All three joints conspired to allow undeviating vibration in a plane at right angles to that allowed by the topmost joint alone, just as though there

had been only two joints at right angles. I also tried three joints at angles of 90° and 30° , and two joints at an angle of 60° . They all gave similar results. It is clear that the planes of slow and quick vibration are determined solely by the position of the topmost joint, the former being that in which the pendulum can swing by the topmost joint alone, and the latter that in which the topmost joint takes no share at all, or the least possible share, at right angles to the first. However the joints may be

arranged, they conspire to establish a virtual second joint, at right angles to the topmost joint, and at some point below it not necessarily coinciding with the position of any one of the actual joints, and apparently varying for different positions of the plane of vibration.

At this stage I showed some of my pendulum's tracery to my brother-in-law, Mr. E. J. Routh, M.A., of St. Peter's College, Cambridge, and received some hints from him which led me to adopt an arrangement whereby the position of the lower joint could be varied, so as to bring the two periods of vibration into any simple proportion, as $1:2$; $2:3$, &c. This was done by a very simple mode of suspension,—two cords hanging from two fixed points some distance apart, and passing through a small ring that could slide up and down the concurrent cords and be fixed at any height. In this arrangement I found the germs of an infinite variety of curious and elegant curves, that gave a new lease of life to my course of experiment. Before attempting anything, I drew in my pocket-book the skeletons of the curves that would be described corresponding to the proportions, $1:2$, $2:3$, $3:4$, $4:5$, $5:6$, $6:7$, $7:8$, $8:9$, in the periods of quick and slow vibration. Then I hung a bullet by two threads from two points about ten inches apart on the circumference of a child's wooden hoop, fixing the hoop upright with the bullet swinging inside the circle. Another bit of thread furnished the sliding-ring to nip the two convergent threads at any desired point, and a few trials enabled me to fix it pretty accurately for the proportion $1:2$, so that the bullet would swing in the plane of the hoop in half the time it took to swing at right angles to the plane of the hoop, making two vibrations in the plane of the hoop to one vibration athwart. (The length of the pendulum varies as the square of the period of vibration, so the slider was at a point one quarter of the distance from the bullet to the horizontal line between the two points of suspension.) It was with great interest that I watched the motion of my pensile bullet, and greeted the verification of my pocket-book sketch. Pull the bullet aside and start it obliquely, and it describes a crescent-moon, the two horns being formed by the double swing in the plane of the hoop, while the length of the crescent measured from tip to tip is given by a single swing athwart the plane of the hoop. Start the bullet from rest at the centre by a sudden blow obliquely given, and it describes a figure of eight with its length athwart the plane of the hoop. (See Figs. 5 and 6.) Other proportions gave still more curious results in accordance with my skeleton sketches, and made me impatient to try them on a larger scale. The lead at my command I packed into a long-shaped zinc box, with a tubular orifice at the top, which in my service became the bottom. To the lead I added a stone jar containing some 10 lbs. of mercury, and made everything secure with cordage. The dependant tubular orifice seemed made on purpose to accommodate a few inches of wooden roller which carried the glass pen, and a diagonal beam in the ceiling of my bedroom offered a capital fixture for two rings about four feet apart, giving suspension to two iron chains by which my incongruous pendulum-bob was doomed to swing. To nip the chains together at the requisite height, I used a loose link which hooked into the corresponding links in the two convergents, and made a very rude and coarse adjustment, which left all accuracy to chance. It chanced, however, that the adjustment for the proportion $2:3$ was beautifully accurate, and I shall never forget the feeling of delight which I experienced while watching the marvellous fidelity with which the pen-point traced the curve appropriate to that proportion. The pendulum was drawn aside and started obliquely on one side of the plane of slow vibration, and having to make three vibrations across that plane to two vibrations to and fro, it compounds these into a curve like a capital Q with two tails, one on each side, looking like a swallow-tailed balloon. (See Fig. 9.)

At the end of the second to-and-fro vibration the pen returns to the point whence it started, except that friction compels it to fall short little by little at every stroke; but if the adjustment is accurate, as it was in this particular case, the shape of the curve remains the same from first to last, and the figure is filled up to the very centre by the orderly description of curve within curve conspiring to produce a web of lines of astonishing regularity. If the adjustment of the connecting link is very slightly inaccurate, the curve begins to change its shape little by little at every stroke, in one way or another, according as the link is too high or too low; and wonderfully intricate is the result, for after a due series of intermediate stages, the original figure reappears, but reversed; and after another series of changes it presents itself again to view in its original posture, but much diminished by the friction that has been in operation throughout all these changeful phases. It may be imagined how intricate the web becomes, though the limits of illustration do not allow me to give a specimen here. It was easy to eliminate all the transition-curves from the tracery, by depressing the paper for the proper interval, and allowing it to return to contact with the pen only at those distinctive phases when the original figure was reproduced either erect or reversed. I obtained a very curious specimen by applying this selective method to the case of the proportion $1:2$, allowing the pen to mark only the crescents and the figures of 8 in alternate series, converging orderly to the centre.

For the suspension of the paper, I fixed four pairs of upright rods at the four corners of a shallow tray, which could be slipped under the pendulum, and each rod gave support to an india-rubber band, which, with its fellow at right angles, was attached by a small hook to the corresponding angle of the paper. Each band could be slid up or down its rod, to allow of nice adjustment of the level of the paper, and the whole tray could be raised on a foot-stool or chair, to suit the elevation of the pendulum when the slider was run aloft in attaining the proportions nearer unity, such as $7:8$, or $8:9$.

The iron chain was soon exchanged for strong cords, passing through a narrow wire ring, which could be arrested at any point by a needle driven through both cords below the ring. This was a small improvement, allowing more accuracy in the adjustment of the slider, and therefore more accuracy in the proportion between the two periods of quick and slow vibration. But it was still very far from satisfactory. Meanwhile I had ordered a cylinder of lead, weighing half a cwt., to be cast, with a hole through the axis; for my zinc box full of "notions" was so tall that I could not bring the slider near enough to the centre of gravity to obtain any proportion lower than $1:3$, and that only with great trouble. When the cylinder of lead appeared, I sawed it into two unequal portions; so that I could use either or both; and instead of simple cords, which twisted in a most troublesome manner below the sliding ring I introduced a stiff rod of fir to carry the lead by a cross pin, and I used two pairs of cords passing through holes in a slider on either side of the central rod. This slider was a small block of wood pierced to fit the rod, and provided with a lateral screw to fix it at any required height. This arrangement ensured admirable steadiness and freedom from torsion, and a great many sheets were filled with the improved performances of the machine; but there still remained an important defect to remedy. The coarse cords, at the point where they entered the holes in the slider, made a very rough hinge for the cross vibration to rely upon, and it was manifest in the tell-tale records of the curves described that considerable change of period took place between the beginning and the end of the web; and the change was always such as to increase the disparity between the two periods, which could only mean that the level of the centre of quick vibration in the cords immediately above the slider was lowered when the range

of oscillation diminished. It was easy to see that a large oscillation would strain the cords to a greater height above the slider than would be called for in a smaller oscillation. The truth of this surmise was proved by the success of the remedy applied. Instead of cords I used two pairs of broad tapes, and instead of a solid slider I made one in two halves, embracing the rod in the centre and nipping the concurrent tapes on either side between their opposed faces, being clamped together by thumb-screws beyond the tapes on either side. Here the slider had no firm hold on the rod beyond the accuracy of its fit, which served to prevent torsion, but had firm hold on the pairs of tapes, pinching them with especial accuracy at the upper edge of the slit in which they lay between the two halves, and reducing the hinge there to a narrow line no thicker than the pairs of tapes instead of the gross thickness of the cords which they superseded. The improvement of the pendulum's performance on paper was very striking. When well adjusted, it was scarcely possible from beginning to end to detect any change in the shape of the figure described; *scarcely* possible, I say, for even now our hinge is not a mathematical line, and we do not obtain perfect mathematical accuracy in our results. Further improvement might be obtained by refinement in tapes and slider, or by increasing the total height of the pendulum, or by substituting some other form of hinge; but the form which I have described is so simple, and its performance so good, that I am content to accept its one very small fault for the sake of its many excellent qualities.

Figures 1-12 are the produce of this pendulum thus improved. They are only a few of the most interesting out of an endless variety of interesting curves, and are chosen as characteristic specimens of a series too extensive to be fairly represented except by a much larger number of illustrations. Figures 1 and 2 represent the proportion 1 : 3, the lowest that is easily attainable without a lofter pendulum; and the following pairs of figures show successively the proportions 2 : 5, 1 : 2, 3 : 5, 2 : 3, 3 : 4. Each of these is illustrated by two figures exhibiting the two chief types of the curve proper to that proportion. They may be termed the cusped type and the looped type. It will be seen that the two cusps in the first figure of each pair are opened into loops in the second, and that each loop in the first is doubled in the second. Between these two typical forms we have an infinite series of intermediate forms possessing features of great interest, those nearest the cusped type especially being characterised by a peculiar "watered" appearance, due to the intersection of two sets of lines very slightly inclined to one another. This is seen, for example, in Fig. 3, which errs a little from the perfect type.

Accuracy of proportion between the two periods of vibration could only be arrived at by repeated trials. The sliding-clamp sufficed for coarse adjustment, but for fine adjustment it was found necessary to attach a subsidiary weight below the large one in some way admitting of considerable range of position, so as to alter minutely the position of the centre of gravity. A heavy iron nut travelling easily on a screw-thread cut on the depending shaft that carried the pen supplied this want, and greatly facilitated the attainment of the utmost accuracy at command.

With a pendulum only seven or eight feet high, there is great difficulty in obtaining the curves that correspond to any proportions lower than 1 : 3, because the slider cannot be brought within a certain distance of the centre of gravity, which lies somewhere in the middle of the lead. To obtain the proportion 1 : 3, that is, to make the pendulum swing three times across for every one swing to and fro, we must lower the slider within a foot of the centre of gravity (the length of the pendulum varying as the square of the period of oscillation), and to obtain the proportion 1 : 4, the distance between the

slider and the centre of gravity must be 1-16th of the height of the pendulum, or only six inches in the present instance; but three or four of those six inches are taken up with the thickness of the lead and the attachments of the tapes, and the rest with the depth of the slider, and so the curve cannot be obtained without a more lofty suspension for the pendulum. This greater elevation I found in the great octagonal room which Sir Christopher Wren built as the chief room of the Royal Observatory in Greenwich Park. By means of two hooks fixed above opposite windows in this room, from which my tapes converged to the middle, I got a height of eighteen feet, and was able to reach such proportions as 1 : 4, 1 : 5, 1 : 6. At this extreme it was really amusing to watch the busy haste of the manifold cross-vibration over-riding the staid gravity that marked the slower oscillation to and fro. To obtain proportions lower yet than these, I should want a great increase in height of suspension; but there is no great inducement to attempt this, as the nature of the curve may be foreseen at a glance, and is marked by extreme simplicity—merely a zigzag or a string of beads.

Some of these experiments with lofty suspension were made on stormy days; and while watching the travels of the delicate pen-point, I could see that their regularity was slightly disturbed by every gust of unusual violence that beat against the high walls.

But this article would never end if I allowed myself to dwell on all the points that called for attention in the course of experiment which I have been describing. I fear I have exceeded due limits already, and feel that I owe an apology to the reader for so large a trespass on his patience. My apology must be the elegance and exquisite symmetry of these natural curves in their admirable obedience to a purely natural law, and the great pleasure I have enjoyed—the sense of high privilege I have felt—in their investigation. I understand that these curves, or some of them, have been demonstrated before, by means of a stream of sand flowing from a hole in the base of a vessel that was used as the weight of the pendulum, and I believe that steel springs of elliptic or oblong cross-section have been made to trace such curves as that which first attracted my attention in the vibration of my slender acacia-twigs; but I am not aware that any specimens of the series have ever before been exhibited in a form that rendered them accessible to the public eye.

HUBERT AIRY

SOME SPECULATIONS ON THE AURORA

IN preparing a lecture on the Aurora Borealis some months ago, I was led to some speculations which may or may not be new, and may or may not be of some value. I will submit them to the readers of NATURE.

I assume of course that the auroral rays extend to great heights above the surface of the earth, that they are sensibly parallel, and that their apparent point of convergence is, generally speaking, that to which the freely-suspended magnet points. In the great aurora of October 24, 1870, this point was close to η Pegasi at 8.30 P.M., coinciding very well with the direction of the magnet.* Remembering that this aurora was witnessed over a large part of the northern hemisphere, and that there was a contemporaneous aurora in the southern hemisphere, and, assuming that at each place the direction of the auroral streamers is approximately parallel to the magnet, we must conceive the earth, during such an auroral display, as a globe with streamers of light radiating and diverging from its polar regions, and spreading far out into space. The general direction of these streamers at different spots on the earth will be got by placing a magnet below a sheet of paper and getting the magnetic curves with iron filings,

* To-night it is a few degrees below α Cygni (but not clearly defined) at 11 P.M.